**MATH 301: FUNDAMENTAL CONCEPTS OF MATHEMATICS**
**COURSE POLICIES AND EXPECTATIONS**
**SPRING 2015**

## 1. Course Information and Goals

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| Course | MATH 301-02: Fundamental Concepts of Mathematics (4.00 Credit Hours) |
| Prerequisites | Six credits of mathematics or a grade of at least B in MATH 102 |
| Instructor | Dr. Mitchel T. Keller, Ph.D., Assistant Professor |
| Email | kellermt@wlu.edu (I generally reply within 24 hours, except on weekends) |
| Time/Location | MTWRF 1325–1525, Robinson 107 |
| Office Hours | MTWRF 1000–1200 and 1530–1600 and by appointment |
| Office | Robinson Hall 206 (540.458.8099) |
| Website | `http://sakai.wlu.edu` |
| Text | C. Schumacher, *Chapter Zero: Fundamental Notions of Abstract Mathematics*.<br>(Second Edition, 2001) Addison-Wesley. ISBN: 0-201-43724-4<br>Burger and Starbird, *The 5 Elements of Effective Thinking*, Princeton, 978-0-691-15666-8 |

**Teaching philosophy.** I believe that the mathematics we understand the most deeply is the mathematics we construct ourselves. Reading a proof someone else has written or watching them present it, even if they cover every excruciating detail, generally only leaves us accepting that we have seen a valid logical demonstration of the statement's truth. That's the way you've approached most of the mathematics you've seen in classes before: "Yes, I feel convinced that this thing you're telling me about really is true, professor. Please don't ask me to explain to you why it is true." However, as you move into 300-level mathematics courses, the level of abstraction will increase and you will be expected to truly understand the details of proofs you read or see in class. You will also be required to produce your own proofs for homework and tests and clearly communicate those proofs in writing. As far as I'm concerned, the only way for you to acquire those skills is by getting a lot of practice.

Solving problems and communicating in the language of mathematics (expressed using English words, but with enough distinct characteristics that we often think of learning to communicate mathematics as similar to learning a foreign language) is best done by doing rather than watching others do. The literature indicates this approach helps students understand the material more deeply and retain it much better. This style of teaching is generally described as "inquiry-based learning" (IBL) and was pioneered in mathematics by topologist R.L. Moore at the University of Texas-Austin. (Moore's full name was Robert Lee Moore, and your suspicion of his namesake is correct.) My personal style encourages much more collaboration amongst students than the "Moore method" as implemented by Moore himself, however. One quote of Moore's does resonate with my philosophy in this course, however: "That student is taught the best who is told the least." I'll spend a lot of time listening to your ideas and asking you questions to help you identify weaknesses in your argument or where to go next, but I will also do my best to avoid telling you much directly. However, I will frequently share my insights as someone with more experience as a member of the mathematical community than you have, and on occasion, you'll even see me present a proof or get to read a complete one in the text.

In this course, I want to help you become better at doing mathematics. I'd like to quote the Hungarian-born mathematician Paul Halmos[1], who wrote in his book *I Want to Be a Mathematician* about his view of what it means to do mathematics:

> Don't just read it; fight it! Ask your own questions, look for your own examples, discover your own proofs. Is the hypothesis necessary? Is the converse true? What happens in the classical special case? What about the degenerate cases? Where does the proof use the hypothesis?

[1]Originator of the shorthand "iff" for "if and only if" as well as the □ symbol to end a proof, which is sometimes called a "halmos" in his honor.

At times, our text specifically asks you to engage in this sort of inquiry, but even when it doesn't, you should do so. (And don't worry if some of the language in that quote seems like technical jargon right now. You'll know what every word means by the end of the first week.)

**Course description and goals.** The official catalog description of Math 301 is "Basic analytical tools and principles useful in mathematical investigations, from their beginning stages, in which experimentation and pattern analysis are likely to play a role, to their final stages, in which mathematical discoveries are formally proved to be correct."

Unofficially, this course could be described as "mathematics boot camp", since after completing this course, you'll have exerienced the process of really **doing** mathematics. Over the course of our four weeks together, we will look at how we can identify patterns that appear to hold with a great deal of generality, how we can prove that those patterns really do hold (or, perhaps, that they don't hold as often as we had thought, but still in some more restrictive scenarios), and how to write mathematics (including typing mathematics using the free LaTeX document processing system). Successful completion of this course will set you up well for other 300-level math courses here, although some may first require completion of Math 221 (Multivariable Calculus) or Math 222 (Linear Algebra).

What can be identified as specific learning goals for this course?

(1) Students will write readable, concise, and mathematically rigorous proofs using standard mathematical English.
(2) Students will identify incorrect proofs and inadequately-supported claims in proofs.
(3) Students will use written mathematical English to express their knowledge of the terminology, concepts, basic properties, and methodology of symbolic logic, set theory, relations and functions, mathematical induction, and other topics selected for the course.
(4) Students will identify correct proof structures and criticize incorrect proof structures.
(5) Students will distinguish between true and false statements about sets, relations, functions, and other topics selected for the course.
(6) Students will discover and explain examples and counterexamples of statements about sets, relations, functions, and other topics selected for the course.

**Course Topics.** The absolute most important goal for this course will be that you do mathematics, including writing proofs and reading proofs others have written to determine if they are complete and correct. However, there are some topics that are standard fodder for helping you through that process. Schumacher's text is called *Chapter Zero* because much of the material in here could be added to the front of any 300-level mathematics text as a "Chapter 0" to make sure students have the necessary background for that course. We will explore most of chapters 1 (logic and proof techniques), 2 (sets), and 3 (mathematical induction). After that, we'll sample key ideas from chapters 4 (relations) and 5 (functions). Since we should still have some time left after that, we'll see what will be of greatest interest and use for students in the class, since after studying the critical key concepts, the main objective is to get practice writing proofs, not learning any particular more advanced concepts.

**Why *Chapter Zero* and *5 Elements of Effective Thinking*?** Carol Schumacher was part of the group that facilitated the workshop where I was first introduced to IBL as a teaching strategy, and I share a lot of her philosophy on how a course like this should be approached. She has years of experience teaching students how to think like mathematicians and write proofs, and *Chapter Zero* is the product of that experience. I particularly enjoy the balance between basic examples and exercises students complete while reading the text and theorems students are expected to prove along the way. Unlike other IBL texts I've used, I find that the hints in this text provide just the right nudge to get students over the hump when a problem or theorem might otherwise be too much of a stretch for students. Of course, like any text, *Chapter Zero* is not without flaws. There are some odious typos, and I'll alert you to them as we approach them. You'll also get to hear me go on a couple of rants about things where I disagree with the author on a topic.

*5 Elements of Effective Thinking* is an easy read that's meant to help you think about how to tackle complex, unfamiliar problems. It presents what the authors claim are the five most important elements of effective thinking along with little thought exercises to allow readers to practice effective thinking along the way. We'll read this quickly during the first week of the term, but themes from it will frequently recur throughout the course. I hope you'll find them recurring throughout your life after the course as well.

## 2. Assessment and Grading

**Marks for written work.** Much of your written work will be marked using the following rubric.[2] Please note that you should *not* simply map these marks onto the A-B-C-D-F letter grades! Later in this document you will find information about the fractions of each of these marks I expect from a student earning particular letter grades.

- E—Exemplary. You have provided a complete, correct solution/response free from errors that demonstrates deep understanding of the question and adheres to the guidelines in our *Elements of Style*. Proper use of LaTeX.
- M—Mastery. You have provided a complete solution/response that demonstrates overall understanding of the question, but that may contain minor errors, modest gaps or omissions, OR (exclusive) minor errors in language/communication/LaTeX usage. (This is the highest grade you can earn on any revision of prior work.)
- P—Progressing. You have some good ideas and have shown significant progress toward a solution that is valid and demonstrates understanding, but there are one or more significant errors in your work or one or more significant issues in the written presentation, or possibly a substantial number of minor issues.
- S—Started. You have begun to respond to the question and have demonstrated time and effort, and possibly some productive failure, but have not yet made significant progress toward a solution.
- I—Incomplete, incomprehensible, or irrelevant. For at least one of these reasons, the work you've provided cannot be reasonably considered a start to the problem.

**Grade components.** Your grade in this course will be based on three categories of work: homework, classwork, and tests. These categories and their respective weights are described below.

**Homework:** Because of the intensive nature of this course, homework will be due every day. Since we are learning to do mathematics as mathematicians do mathematics and modern mathematics is done in a very collaborative manner, you will write up this homework in groups. I will assign you to a group of two or three students. We'll change up the groups each Monday. (Homework due on Mondays will be done with the group you worked with the previous week.) All homework will be typed using LaTeX and stored on Box. I will give you feedback via comments on Box as well. (See additional handout on Box logistics for more information on this.) You are allowed (Encouraged, in fact!) to collaborate with other groups, too. However, each group must write up its solution/proof independently of the other groups. That means you can take scratch work or photos of board work you did together and work from that, but you cannot take the written/typed work of another group and use it as the basis for your written homework.

Each homework problem will be marked using the E-M-P-S-I rubric above. The table below gives you a rough guide to how the total number of E's, M's, etc. you earn on homework will translate into your homework grade, which will be 30 percent of your course grade.

| Grade | Criteria |
|---|---|
| A | No S's or I's, almost no P's, majority E's |
| B | No S's or I's, limited (at most 1/5) P's, some E's (at least 1/5) |
| C | No I's, limited S's (at most 1/10), at least 2/3 M or higher |
| D | No I's, vast majority (3/4) P or higher, significant number (2/5) M or higher |
| F | Does not meet above standards |

I will also check these expectations on a chapter-by-chapter basis to ensure a solid distribution of understanding across the course topics. As stated above, this table is a guideline only. In particular,

[2] This rubric and the letter grade descriptions later in this document are based on materials from Matt Boelkins of Grand Valley State University.

the fractions included are estimates of how I would interpret the words I've used to describe each level. Your grade will be determined based on my best professional judgment of your achievement.

**Classwork:** Your classwork grade is worth 30 percent of your course grade. Classwork grades will consider your preparation for each class, class participation, presentations, and productive failures. Preparation will be measured by tracking what work you have done on the assigned material from the text when you come to class. Notes you take during class will be written using a felt-tipped pen I provide so I can distinguish what you did before class. I'll mark your preparation on a ✓-system at the end of class each day. Class participation includes your work with your group during classtime as well as asking and answering questions during times where the whole class is discussing something or listening to a student presentation.

Most days, we'll have a couple of problems that students will present to the class. We're aiming for a friendly, collegial environment in this class, but since student presentations are a key element of this class, we must take them seriously. Here are some of my expectations for students (both presenter and audience) during student presentations[3]:

- In order to make the presentations go smoothly, the presenter needs to have written out the solution/proof in detail and gone over the major ideas and transitions, so that he or she can make clear the path of the argument to others.
- The purpose of class presentations is not to prove to me that the presenter has done the problem. It is to make the ideas of the proof clear to the other students. We will generally not move on until the class agrees that a complete and correct solution/proof has been presented, although sometimes we will defer the correction of a solution/proof until a future class.
  **Important**: Do not be shy about asking questions of presenters. If something is not clear to you and everyone is silent, do not assume that the silence means you are the only one wanting clarifiation. It is more likely *everyone* wants clarification about something. You are not "holding the class up" by asking a question (or six).
- Presenters are to write in complete sentences, using proper English and mathematical grammar. English sentences and mathematical grammar allow for the proper use of notation and symbols. However, since this is not a course in formal logic, the symbols for quantifiers ($\forall$ and $\exists$) should be avoided outside of scratchwork, which is not to be presented or turned in as homework. (The exception would be if the whole point of a problem is to do something with quantifiers.)
- The board at the front of R107 will be designated as our presentation space. For each class, I'll ask for volunteers for the problems I've chosen for presentation and select the presenters. While the other students work in groups on something, the presenters will write their proofs on the front board. When I decide the time is right in class, they will then take turns presenting. A presentation entails more than just reading aloud what is written on the board. It should include additional comments about why something follows from a previous fact or what inspired the decision to follow a certain path.
- Fellow students are allowed to ask questions, but questions **must** be held until after the presenter finishes. At that point, we'll pause to allow everyone (Me included!) to soak in the presentation we just saw and formulate comments, questions, or compliments for the presenter. We'll then have time for sharing some of those comments, questions, or compliments. It is the responsibility of the person making the presentation to answer questions to the best of his or her ability, and other students should not answer on behalf of the presenter unless I indicate it is OK to do so.
- Since the presentation is directed at the students, the presenter should frequently make eye-contact with the students in order to see how well the other students are following the presentation. Confused looks from the students should be interpreted as a silent request that you give more detail in your explanation.

[3]Adapted from Dana C. Ernst's presentation expectations, which were in turn derived from the *Chapter Zero Instructor's Resource Manual*.

- Hopefully most of the questions will come from students in the class, but occasionally I will have a question to ask as well. When I ask a question, however, the presenter is *not* allowed to answer. Instead, I will call on someone not at the board and ask them to answer my question. (I'll keep track of to whom I've asked questions when I do this to balance out my questioning, but I won't be grading your responses per se.)
- Once the class seems content with the presentation, I'll try to make some comments about style and any potential issues for written homework submission of the proof, especially if there are any things we clarified orally but didn't get written on the board. You are invited to ask questions of me about mathematical exposition at that time.
- If a significant flaw is found in a presenter's proof, we will "pause" the presentation and allow that student to return either later in the class or in the next class to provide a corrected version of the proof. (The presenter is encouraged to consult with his/her group and me before the next class to remedy the flaw.) However, do not volunteer to present if you have not spent time thinking about the problem and organizing your thoughts in written form before class.

The other component of classwork is presentations of *produtive failures*. What's a productive failure? It's a moment where you made a mistake in the process of solving a problem in this course, but that mistake (failure) led you to a better understanding of a topic or problem (or even your own strengths and weaknesses). It might also be a trial and error approach to solving a problem where figuring out why something didn't work (failed) helped you see what would work. Why give you credit for sharing these moments? Because human beings, from their earliest days until their last, learn by making mistakes. Celebrating success should also include celebrating turning a failure into something valuable for the future. (This is a topic that we will see in *5 Elements of Effective Thinking*.) I expect each student to (briefly) share (at least) one productive failure over the course of the term. Whenever you've got a productive failure to share, just put "PF" and your initials on the presentation sign-up, and I'll save time for you to share during that class. (Given time constraints, sharing productive failures make take place between two groups instead of to the whole class occasionally, but we'll aim to have a couple minutes for a very quick discussion of your productive failures most days.)

Your grade for classwork will be determined based on my best professional judgment of your contributions and achievements, but I do have some minimum standards to earn a C or higher:

- at least 2 successful presentations (A flawed presentation that is fixed in the next class counts as a single successful presentation.),
- at least 1 productive failure presented,
- regular (not necessarily daily) contributions to class discussions and presentations,
- contributions to group work in class on a daily basis, and
- come to class daily with serious work on the majority of assigned tasks from the text (✓).

Earning a higher classwork grade will involve some combination of more presentations, really strong presentations, insightful contributions to class discussions/presentations, full engagement in group work during class, and coming to class with very complete work for the tasks assigned from the text (✓+). I *expect* everyone in the class to get an A or B for classwork, and I will let you know if I feel you need to step up your game if you don't want to endager this part of your grade. However, don't hesitate to ask me if you're worried about where you stand in this regard.

**Tests:** After each week, we will have a self-timed, closed book, closed notes take-home test. I'll designate the time limit for each test, but it will typically be 90 minutes or 2 hours. (There will be no comprehenstive final exam for this course. You should use whatever time period you wish on Saturday, 23 May 2015 to take Test IV.) The logistics for each test will be described closer to the administration of the first test, but each test will have three principal components:

- Definitions: In mathematics, we must know definitions fully and be able to use them. You will be building a LaTeX file full of definitions to study from (and use in future courses) as the course progresses. For each test, I'll tell you a list of definitions you're responsible for knowing. On the test, I'll ask you for some of those definitions. Definitions will be graded as correct or incorrect, since a half-correct definition is incorrect and of no mathematical use.

- Proofs: These will be new problems for you to prove, but they will be similar to things you've done in class or for the homework. I will give you some guidance in advance of each test as to the types of problems that will appear for proofs. There will be more things available for you to prove than you are required to prove.
- LaTeX task: I will provide some small piece of text or math that I have typeset using LaTeX. You will be required to figure out what the LaTeX code that produces that piece of text or math is. This is the one part of the test where you may use your notes, text, the Internet, etc. (but you may not collaborate).

As with the other parts of the course, your grade on the tests will be determined by my best professional judgment. Each test is 10 percent of your course grade. Here are some general guidelines as to how I will grade the tests. Before you take each test, I'll be able to give you a greater amount of specificity as to the number of definitions I'll require, the number of proofs that will be available for you to choose from, and the minimum number required. (Failure to do the minimum number will result in a grade of I on as many proofs as are required to have assigned a mark to the minimum number.)

| Grade | Criteria |
|---|---|
| A | Full accuracy on definitions. Does more than the minimum required number of proofs with all grades M or higher or does the minimum required number of proofs with all grades M or higher and several E's. LaTeX task completely correct. |
| B | At most 1 incorrect definition. Grade of M or higher on minimum required number of proofs. LaTeX task essentially correct, perhaps with a minor flaw. |
| C | At most 1 incorrect definition. Grade of M on most proofs and no grade lower than P. LaTeX task is mostly correct, but perhaps with a more serious flaw. |
| D | At most 2 incorrect definitions. Grade of S on at most one proof (of the minimum number required) and a grade of P or higher on as many as are needed to get to the minimum number required. LaTeX task is mostly correct, but perhaps with a more serious flaw. |
| F | Fails to meet the above standards. |

**What does a course letter grade mean?** I've mentioned a few times that your final grade in this course will be based on my best professional judgment of your accomplishments. In addition to the more quantitative information provided above, here's a rough qualitative guide to what I'm thinking about when assigning final course letter grades.

| Grade | Description |
|---|---|
| A | Deep understanding of the vast majority of the key ideas in the course; able to solve challenging problems with minimal input from the instructor; excellent written and oral skills in communicating mathematical ideas; strongly prepared to move on to another proof-based math course. |
| B | Good understanding of a significant majority of the key ideas in the course; able to solve challenging problems with modest support from the instructor; solid written and oral skills in communicating mathematical ideas; well prepared to move on to another proof-based math course. |
| C | Basic understanding of a majority of the key ideas in the course; able to solve challenging problems with significant support from the instructor; satisfactory written and oral skills in communicating mathematical ideas; minimally prepared to move on to another proof-based math course. |
| D, F | Not ready to move on to another proof-based math course because of insufficient understanding of key ideas in the course, limited ability to solve challenging problems independently, or lacking adequate skills in communicating mathematics orally or in writing. |

**How am I doing in this class?** This course will move at a very fast pace, so it's easy for a small setback to snowball and leave you far behind. Thus, I encourage you to check in with me if you're curious or concerned about how you're doing grade-wise in the course. The best way to have a conversation like that with me is to let me know what grade you're aiming for in the course, and then I can advise you as to whether you're on track for that grade if you continue as you've been doing or what areas require further effort.

**Make-up tests and late work.** Given the pacing of this course, late work is generally unacceptable and typically will not be accepted. If you must miss class, please discuss it with me in advance. Tests will be self-timed and take-home, so late tests will not be accepted.

## 3. Course Policies

These policies and the course schedule are subject to change depending on how the semester progresses. Any changes will be announced in class, on the online course announcements, and with an updated version of this document on Sakai. The update date will be clearly indicated in the document's title.

**Attendance:** Because student presentation and class participation are the cornerstone of this class and happen only during class meetings, attendance is vital and will be recorded for classwork grade purposes.

**Being prepared for class:** Each day, you should bring your copy of *Chapter Zero* to class, as we will frequently do activities that involve you using the text with your group. Also bring your notes/work from the reading the night before. You may wish to have your computer handy to consult your prior homework or definitions file, since you might not have all those things in your notes.

**Technology use in class:** Other than times where class activities require them, I discourage use of laptops, smartphones, or other electronic devices during class. Students are expected to discontinue use of electronic devices if the instructor or another student deems them distracting.

**Noise from telephones, etc.:** Audible noises from cellular telephones and computers are distracting to me and your classmates. This includes the noise made by some models when set to vibrate. Please turn your phone off or set it on silent before class begins. I reserve the right to take actions that may embarrass you if your phone rings in class.

**Record retention:** You should retain all graded materials returned to you until after final grades have been posted. You will need them to support any claim that your grade was inaccurately computed.

**Collaboration.**

- Please collaborate with other students in this class on both problems you're doing to prepare for class and the written homework. (In fact, you are required to collaborate on the written homework.) You will learn better if you do so. You are also free to ask me questions during office hours, by making an appointment to see me, or by catching me when it looks like I have a spare moment.
- Even though you will collaborate, I expect that each group write up your solutions/proofs independently. Feel free to take notes/scratchwork from your collaboration sessions, but then go off and write up your own homework to submit. Use the LaTeX `\thanks` command with one of your group member's names in the `\author` command to acknowledge any groups with which you collaborated on homework problems that are written up for grading.
- You are **not** allowed to ask anyone other than your classmates and me about the problems for this course. I have specific goals about helping you learn to prove things, and so I know what sort of help best fits with those goals.
- No resources (books, websites, movies, journal articles, etc.) other than the text, your own solutions/proofs to earlier problems, and notes you've taken in class, in meetings with me, or in collaboration sessions with your classmates may be consulted when working on homework or exams. (You may, however, use whatever texts and websites you want to figure out how to do something in LaTeX.)

- For the tests, all work must be done individually and under the time restrictions I have established.

## 4. University Policies

**Academic Integrity.** As we are all aware, the entire W&L community takes academic integrity very seriously. The community of trust is part of what drew me to a career at W&L, and presumably for most of you, it was also a factor in your selection of a university to attend.

If this document and the instructions for graded work do not make my policies clear, please ask for clarification. The most important rules are that the only human resources you may consult (via any medium) for homework are the other students in Math 301 this term and me and that you may not consult any resources other than our textbook, additional notes/handouts I supply, and your own earlier work for this course when doing homework. Tests will be individual and closed book/notes. You may, however, use books and the Internet to assist you in solving the LaTeX problem that is part of each test.

Behavior that violates our community's standards of trust will not be tolerated and will be reported to the Executive Committee. If you are uncertain of my rules or expectations at any time, ask for clarification. Note that it is possible for me to determine that an action is in violation of my policies even if the EC determines it is not an honor violation. In such instances, I may assign a lower grade on the work in question.

**Special needs.** Washington and Lee University makes reasonable academic accommodations for qualified students with disabilities. All undergraduate accommodations must be approved through the Office of the Dean of the College. Students requesting accommodations for this course should present an official accommodation letter within the first two weeks of the term and schedule a meeting outside of class time to discuss accommodations. It is the student's responsibility to present this paperwork in a timely fashion and to follow up about accommodation arrangements. Accommodations for test-taking should be arranged with the professor at least a week before the date of the test or exam.

**Incomplete grades.** Incomplete grades will be assigned only under extraordinary circumstances in accordance with the W&L Catalog, which states "A grade of Incomplete signifies that, due to some cause beyond the student's reasonable control (e.g., illness, injury, incapacitation), the work of the course has not been completed or the final examination has been deferred." If you feel an incomplete grade may be warranted, you must contact me *before* the end of the final examination period at 1700 on Saturday, 23 May 2015.