

MATH 1501G1/G2/G3 Test I

Fall 2006

Name: _____

GTid (9xxxxxxxx): _____

Instructor: Mitchel T. Keller

There are 5 questions on this exam on one page (not counting this coverpage). **Answer each question on a separate solution sheet.** Be sure to explain your answers, as answers that are not accompanied by explanations/work may receive no credit. **Use complete sentences wherever possible;** answers that do not contain at least one complete sentence of explanation (and do not just ask for a list or for you to label something) will not receive full credit. Place your name and problem number(s) on each solution sheet. Any solution sheet missing any of this information will **not** be graded.

You are to complete this exam completely alone, without the aid of notes, texts, calculators, cellular telephones, personal digital assistants, or any other mechanical or digital calculating device.

By signing on the line below, you agree to abide by the Georgia Tech Honor Code and Student Code of Conduct, the principles of which are embodied by the Challenge Statement:

I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech community.

Failure to sign this cover page will *not* be considered evidence of academic misconduct. However, **if the cover page is not signed, fifteen points will be deducted from your raw total score on this exam.**

Student signature: _____

1. (5 points) Using the ε, δ method, show that

$$\lim_{x \rightarrow -1} (-4x + 2) = 6.$$

2. (5 points) Consider the sequence a_n defined by

$$a_n = \begin{cases} 2^{-(n+1)/2} & n \text{ odd;} \\ 3^{-n/2} & n \text{ even.} \end{cases}$$

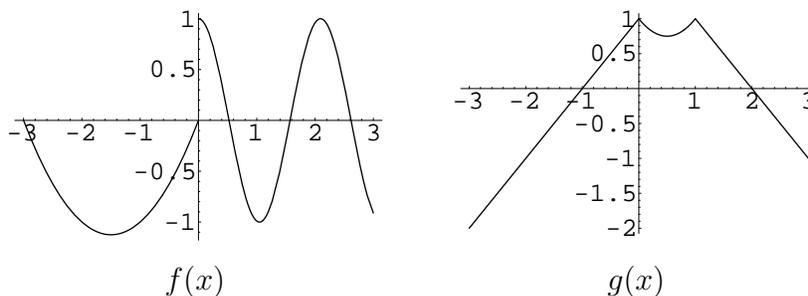
Is the sequence monotonic? If so, what type of monotonicity does it exhibit? If not, why is it not monotonic? Does the sequence a_n converge? If so, state its limit and explain why that is the limit. If not, explain why it diverges.

3. (5 points) Determine if the limit

$$\lim_{t \rightarrow 0} \frac{t^2 \sin(3t) + 6t^4 - t^3 \cos(2t)}{\tan(3t)(1 - \cos(at))}$$

exists. If it does, find its value, and if it does not, explain why. (Assume that $a \neq 0$.)

4. (5 points) Consider the functions pictured below. (Take $f(0) = 1$.)



For what values of x is the function $f(g(x))$ continuous? For what values of x is the function $g(f(x))$ continuous? Justify your answers.

5. (5 points) Show that there is a number $c \in [1, 4]$ such that

$$\csc\left(\frac{\pi}{3}c\right) = \frac{c^2}{\sqrt{3}}.$$